w↑?

How do you write equations to describe the motion of a point in a plane?

10.6 Parametric Equations

Definitions/Characteristics

- Introduce a 3rd variable- called the parameter – represents a curve in the plane.
- A common parameter used is time (t) or an angle (trig)
- (x, y) is the place, "t" is the time it is there (at that place)

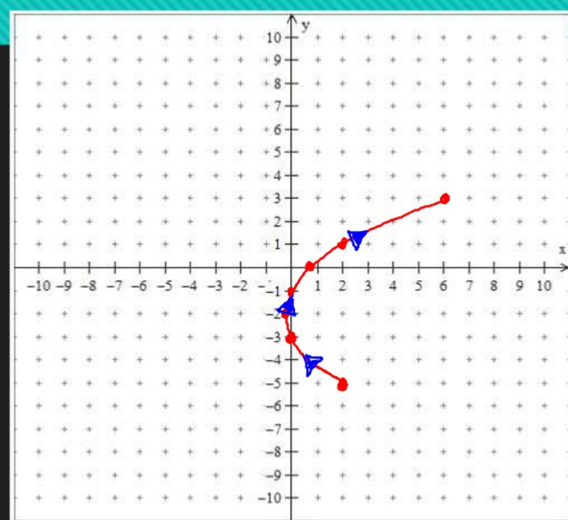
Graphing Parametric Equations

- 2 OPTIONS:
  - (1) Use a chart to find rectangular points.
    Plot the points.
    Show the orientation (flow) by arrows
  - (2) Convert to a rectangular equation and graph it

Graphing Using a Chart

$x = t^2 + t$ $\qquad\qquad$ $y = 2t - 1$

t	X	y
-2	2	-5
-1	0	-3
$-\frac{1}{2}$	$-1/4$	-2
0	0	-1
$\frac{1}{2}$	$3/4$	0
1	2	1
2	6	3

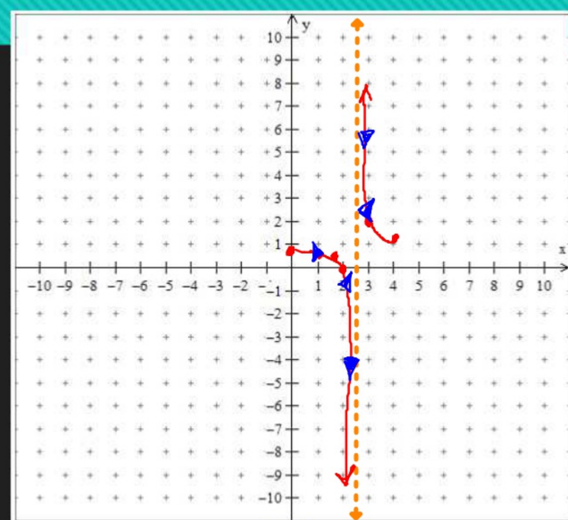


Arrows for Orientation

Graphing Using a Chart

$$x = t + 2 \qquad y = \frac{2t}{2t - 1}$$

t	x	y
-2	0	4/5
-1	1	2/3
$-\frac{1}{2}$	1.5	1/2
0	2	0
$\frac{1}{2}$	2.5	U
1	3	2
2	4	4/3



Converting from Parametric to Rectangular (Eliminating the Parameter)

- Steps:
  1. Solve for the parameter
  2. Substitute the parameter into the 2nd equation
  3. Simplify

Examples: Parameter is time

1.) $x = t + 4$, $y = t - 7$

① $x = t + 4$

$\boxed{t = x - 4}$

② $y = t - 7$

$y = (x - 4) - 7$

$\boxed{y = x - 11}$

2.

① $y = \frac{t}{2} \cdot 2$

$t = 2y$

② $x = t^2 - 4$

$x = (2y)^2 - 4$

$\boxed{x = 4y^2 - 4}$

2.) $x = t^2 - 4$, $y = t/2$

① $x = t^2 - 4$

$x + 4 = t^2$

$t = \pm\sqrt{x + 4}$

② $y = \frac{t}{2}$

$\boxed{y = \frac{\pm\sqrt{x+4}}{2}}$

Examples: Parameter is time

3.) $x = \sqrt{t} + 5$, $y = 2t - 7$

① $x = \sqrt{t} + 5$

$x - 5 = \sqrt{t}$

$t = (x-5)^2$

② $y = 2t - 7$

$\boxed{y = 2(x-5)^2 - 7}$

4.) $x = \frac{1}{\sqrt{t+1}}$ $\quad y = \frac{t}{t+1}$

$x > 0$

$t > -1$

$(\sqrt{t+1})$

① $x = \frac{1}{\sqrt{t+1}} \cdot (\sqrt{t+1})$

$(x\sqrt{t+1})^2 = (1)^2$

$\frac{x^2(t+1)}{x^2} = \frac{1}{x^2}$

$t + 1 = \frac{1}{x^2} - 1$

-1

$t = \frac{1}{x^2} - 1$

② $y = \frac{t}{t+1} = \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} \cancel{-1+1}}$

$\frac{\frac{1}{x^2} - 1\left(\frac{x^2}{x^2}\right)}{\frac{1}{x^2}} = \frac{\frac{1-x^2}{x^2}}{\frac{1}{x^2}}$

$\frac{1-x^2}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{1} \rightarrow \boxed{y = 1 - x^2}$

$x > 0$

Examples: Eliminating the Angle Parameter

1.) $x = 3cos\theta, y = 4sin\theta$

① Solve for $\cos\theta$ and $\sin\theta$

$x = 3\cos\theta \qquad y = 4\sin\theta$

$\cos\theta = \frac{x}{3} \qquad \sin\theta = \frac{y}{4}$

② Use identity $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{16} = 1}$$

2.) $x = 2cos\theta, y = 2sin\theta$

$\frac{x}{2} = \cos\theta \qquad \frac{y}{2} = \sin\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$\left(\frac{x^2}{4} + \frac{y^2}{4} = 1\right)4$

$$\boxed{x^2 + y^2 = 4}$$

Converting from Rectangular to Parametric

- Must be given the parameter
- Steps:
  1. Plug the given parameter into the rectangular equation
  2. Simplify

Examples

Given: $y = 1 - x^2$

Find the parametric equations given the following parameters:

1.) $t = x$

$\boxed{x = t}$

$y = 1 - x^2$

$\boxed{y = 1 - t^2}$

2.) $t = x + 3$

$\boxed{x = t - 3}$

$y = 1 - x^2$

$y = 1 - (t-3)^2$

$y = 1 - (t^2 - 6t + 9)$

$y = 1 - t^2 + 6t - 9$

$\boxed{y = -t^2 + 6t - 8}$

3.) $t = 1 - x$

$t = 1 - x$

$t - 1 = -x$

$\boxed{x = 1 - t}$

$y = 1 - (1-t)^2$

$y = 1 - (1 - 2t + t^2)$

$y = 1 - 1 + 2t - t^2$

$\boxed{y = -t^2 + 2t}$

Practice Problems

- ~~Pg 776-777 # 3-14, 17, 18, 37-44~~

Classwork: Textbook page 776-7 #3-13 odd, 17, 37-43 odd

Homework: #4-14 even, 18, 38-44 even